CNT-104S

Calibration of Time Interval and compensation for systematic errors

APPLICATION NOTE

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Background

CNT-104S is a Multi-channel Frequency Analyzer / Time Interval Analyzer, capable of simultaneously measuring 4 parallel input signals. One of the Measurement functions is the traditional Time Interval, with a timestamp resolution of <7ps. This application describes the uncertainty elements of Time Interval measurements, plus a section on calibration methods to eliminate the systematic channel-to-channel skew.

1. Time Interval Uncertainty elements

A time interval measurement by any timer/counter has a certain uncertainty, varying by model. The uncertainty elements are traditionally split up in

- Random uncertainties that can be reduced by averaging
- Systematic uncertainties that can NOT be reduced by averaging

Random uncertainties

The two error sources are Quantization Error (Resolution) and Noise Trigger error

Resolution is the ability to distinguish how close two values can be and still be recognized as separate values. This is normally defined as the instrument's own spread of values (standard deviation), when you measure a very large number of samples of a fixed, rock-stable, Time Interval.

Since a time interval has one start trigger and one stop trigger, it is common to refer the concept of resolution of the instrument to the spread in each trigger point, called resolution per timestamp. The resolution of the total Time Interval measurement, involving both the spread of start trigger and the stop trigger is found by multiplying the resolution/timestamp with √2 (≈1.4).

In the Pendulum CNT-104S, that resolution/timestamp is <7ps rms, and resolution for a complete Time Interval measurement is <10 ps rms.

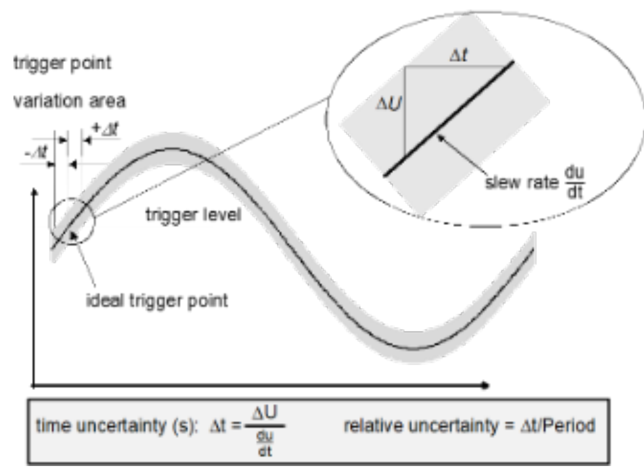
When you measure the same time Interval over and over again, you can use the built-in statistics to reduce random uncertainty, with the square root of the number of samples.

E.g. when using 100 samples, the resolution uncertainty is reduced 10 times, and with one million samples it is reduced 1000 times

The resolution of a repetitive Time Interval measurement can always be reduced to virtually zero by a sufficiently large number of samples.

The *noise trigger error* caused by external or internal noise is the other random uncertainty element. It will be dominant for signals where the trigger point has a low slew rate, for example low frequency sine waves.

Noise uncertainty



Example 1: A 100 Hz, 1Vrms, sine wave with a white noise of 0.5 mVrms from external and internal noise sources will contribute with a time uncertainty in the order of 1 µs, compared to the resolution uncertainty of 7 ps.

Example 2: A 0V to 5V pulse with 4 ns rise time with the same amplifier noise of 0.5 mVrms as above, will contribute with a time uncertainty of approx. 0.5 ps rms, compared to the resolution uncertainty of 7 ps.

Also this random trigger noise error can be reduced by averaging, just like the resolution uncertainty

For a fast rising pulse (<5 ns rise time), with a high slew rate (>1V/ns), the noise trigger error will be one order of magnitude lower than the resolution uncertainty and can be neglected.

Systematic Uncertainties

There are three types of systematic uncertainty elements in Time Interval measurements; Timebase error, trigger level setting error, and channel-to-channel offset error.

The timebase error, mainly due to ageing and temperature effects, causes a systematic error in the time interval measurement, that is noticeable for very long time intervals and totally neglectable for short time intervals.

Example 1: A TCXO reference oscillator may have an error of <1 ppm (10^{-6}), which corresponds to a systematic error of 1 µs on a 1 second Time interval, but only with 1ps on a 1 microsecond interval.

Example 2: An ultra-stable OCXO reference oscillator may have an error of <10 ppb (10^{-8}), which corresponds to a systematic error of 10 ns on a 1 second Time interval, but only to 10 fs on a 1 microsecond interval.

For short time intervals (<1 µs), the timebase error is neglectable for all reference oscillators, and for an Ultra-stable OCXO, the timebase error is neglectable for time intervals <100 µs.

The trigger level setting error occurs when the actual trigger level differs from the intended trigger level. E.g., a Pulse width measurement should always be between the 50% points of the amplitude on the rising and falling edges. Although the hysteresis band in the CNT-104S' input is approx. 20 mV, which would cause a trigger level offset of 10 mV in traditional counters, the CNT-104S features automatic hysteresis compensation, to assure that the actual trigger point deviates typically only 2 mV from the set trigger level. This uncertainty is found in both start and stop trigger points. Once again, this error component can be neglected for fast rising pulses.

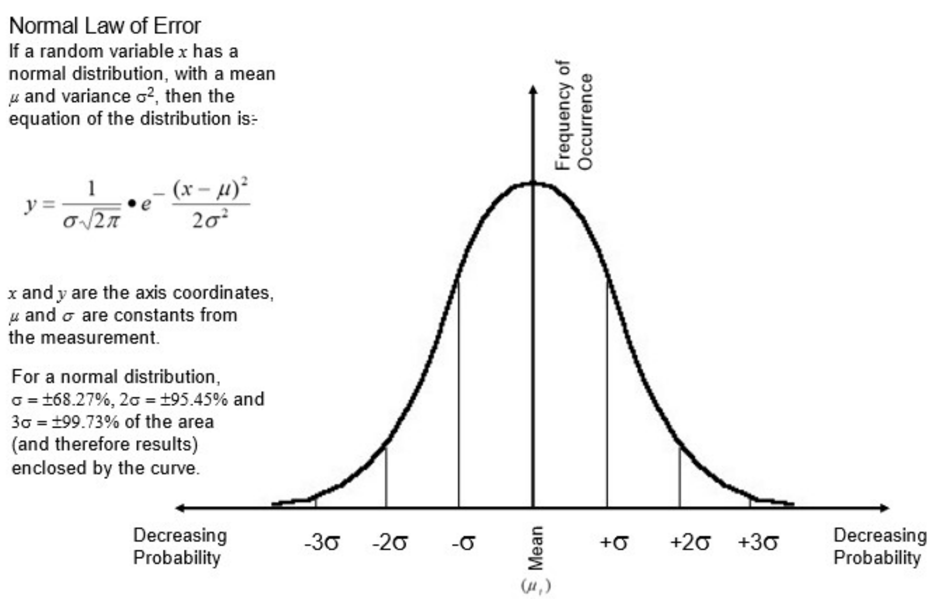
Example: A 0V to 5V pulse with 4 ns rise time will contribute with a time uncertainty of approx. 2 ps.

For a fast rising pulse (<5 ns rise time), with a high slew rate (>1V/ns), the trigger level setting error can be neglected.

The systematic channel-to-channel offset error is typically 100 ps rms. This measurement uncertainty is due to the difference in travel time from the input connector to the actual internal timestamping of the trigger event between the start and the stop channel. That signal path comprises the input amplifiers travel time, delay in the comparators in the channels, and also internal delays inside the FPGA up to the time stamping circuitry.

The channel-to-channel offset is a fixed value for any specific measurement, but varies between measurements due to input amplifier settings, time interval value, input signal frequency, and input signal amplitude. It is internally calibrated in CNT-104S, with some standard signals, and has a residual uncertainty of 100 ps rms.

100 ps rms means that 68% of all offsets are inside ±100 ps (1 sigma), and 95.5% of all offsets are inside ±200 ps (2 sigma)



The channel-to-channel offset error, is the dominant systematic error factor for short time intervals between pulses (<1 ms), but neglectable for long time intervals (>10 ms).

Conclusion: Although we have identified 5 different error sources for the Time Interval uncertainty, most of them can be eliminated for some normal use cases.

General: Random uncertainty can be averaged to very small values and neglected compared to systematic uncertainties

Random uncertainties:

Fast rising pulses: Forget about noise trigger error.

Low frequency sine waves: Noise trigger error dominates. Forget about resolution.

Repetitive pulses: Always forget about resolution, that can always be reduced by averaging.

Systematic uncertainties:

- Very long time intervals: Forget about the channel-to-channel offset. Timebase error dominates.
- Very short time intervals: Forget about the timebase error. Channel-to-channel offset dominates
- Fast rising pulses: Forget about Trigger level setting error.
- Slow rising pulses or low frequency sine or triangular waves: Trigger level setting error will dominate.

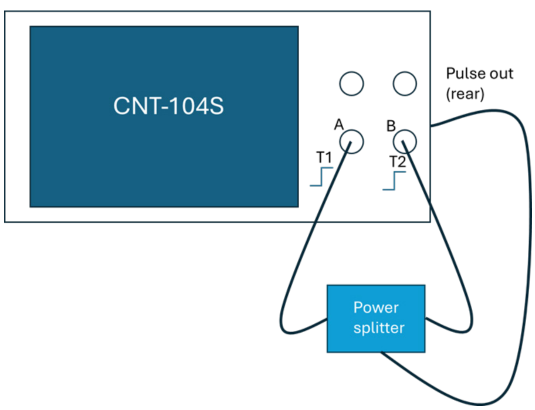
2. How to calibrate the systematic channel-to-channel offset

The channel-to-channel offset error of typically 100 ps rms, cannot be reduced via averaging, but it can be accurately calibrated and compensated for.

Method 1: Send a known time interval of 0 ns to the counter's input and read the result, and compensate thereafter the measured time interval via MATH function.

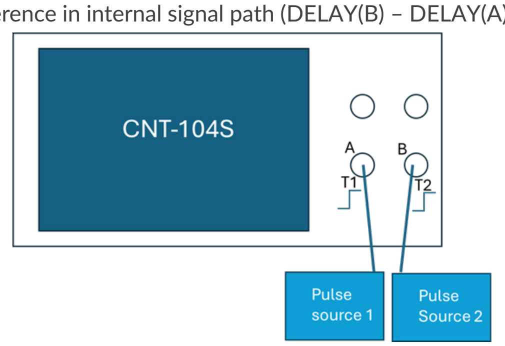
Note: The amplitude and repetition rate of the calibration signal should be as close to the actual signal in the final measurement. Input amplifier parameters may not be changed.

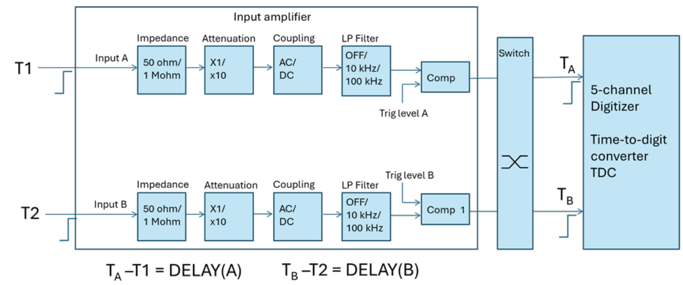
The simplest way to generate a time interval of (close to zero) is to connect a fast rising pulse train with the same frequency as the final test frequency, to a power splitter, and thereafter feed input A and B with cables of the same length. For simplicity you can use the pulse output in CNT-104S.



This method is straightforward and simple but has a basic uncertainty in the travel time through the power splitter and cables. That uncertainty is in the order of 10-20 ps, considered that a cable length difference of only 2 mm corresponds to 10 ps difference in travel time. Furthermore, the CNT-104S amplitude may differ from the actual pulse sources in the test, and a zero-interval calibration may be very accurate for short time intervals but not for a long time interval.

Method 2: Measure your unknown time interval (T2-T1) at the counter's input and read the result. That means your display reading is the combined result (TB – TA) of the unknown time interval (T2-T1) and the difference in internal signal path (DELAY(B) – DELAY(A)).





You know that the uncertainty is typically 100 ps. The next step is to switch the cables externally, which should theoretically result in the same value but with the opposite sign, if there were no internal channel-to-channel offset. E.g. a Time Interval reading of 1.5 ns should in an ideal counter result in a reading of -1.5 ns after switching the cables.

But due to the internal channel-to-channel offset, the reading after switching cable will be different. You will find a *corrected Time Interval value by subtracting reading 2 from reading 1, and divide by 2:*

Reading 1:

TB – TA = (T2 + DELAY(B)) – (T1+DELAY(A))

Reading 2:

TB – TA = (T1 + DELAY(B)) – (T2+DELAY(A))

Reading 1 – Reading 2 =

= (T2 + DELAY(B)) – (T1+DELAY(A)) – [(T1 + DELAY(B)) – (T2+DELAY(A))] =

= T2 – T1 – T1 + T2 = 2*(T2 – T1)

Thus the correct calculated time interval is
[(Reading 1 – Reading 2)]/2.

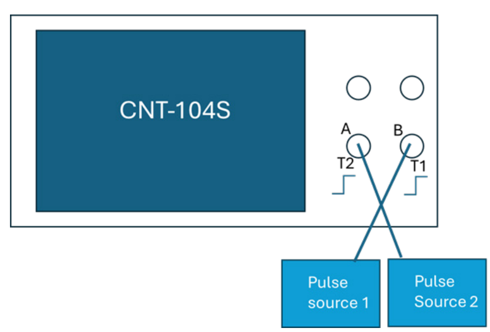
The *systematic error*, DELAY(B) – DELAY(A), is thereafter found as:

Reading 1 – calculated (T2 – T1) =

= (T2 + DELAY(B)) – (T1+DELAY(A)) – (T2 – T1) = DELAY(B) – DELAY(A)

Alternatively the systematic error is found as the average of reading 1 and 2:

Systematic offset error is [(Reading 1 + Reading 2)]/2



Example 1 (approx. 10ns difference); first reading is 10.250 ns, second reading (after swapping cables) is -9.950 ns.

The calculated time interval is

[10.250-(-9.950)]/2 = 10.100 ns

The systematic error is then:

[(reading 1) - (calculated T.I.)] = 10.250 – 10.100 = 150 ps

You can thereafter compensate the measured time interval via MATH function.

- Select MATH function K*X
- set K = 1
- set L = -150 ps.

Example 2 (approx. 0ns difference); first reading is -248 ps, second reading (after swapping cables) is -68 ps.

The correct calculated time interval is

[-248-(-68)]/2 = -180/2 = -90 ps

The systematic error is then:

[(reading 1) - (calculated T.I.)] = -248 + 90 = -158 ps

You can thereafter compensate the measured time interval via MATH function.

- Select MATH function K*X
- set K=1
- set L = +90 ps.

NOTE. The systematic error, besides the timebase error, include the trigger level timing error and the travel time difference from input to digitizer in the FPGA. This calibration method will compensate for both these error sources.

There is a small residual error. But you should expect that to be <10 ps.

Uncertainty example 1 using CNT-104S:

Time interval of approx. 1ns

Pulse output repetition rate 1MHz, amplitude 2.5V and rise time 2.5 ns.

CNT-104S has standard TCXO (uncertainty 1ppm) and 10000 samples average for calibration.

External plus internal input white noise is assumed to be 0.5 mV rms.

Random uncertainty

Resolution: 10 ps rms (single shot), 100 fs rms (averaged over 10000 samples)

Noise trigger error: 0.9 ps rms (single shot), 9 fs rms (averaged over 10000 samples)

Systematic uncertainty

Timebase error: <1 fs (1ns * 1ppm)

Trigger level timing error: <18 ps without calibration, <10 ps with calibration

Channel-to channel offset: 100 ps rms without calibration, <10 ps with calibration.

TOTAL Uncertainty is in the order of 10ps after calibration.

Uncertainty example 2 using CNT-104S:

Time Interval is approx. 100 µs

Pulse repetition rate 1kHz, amplitude 2.5V and rise time 2.5 ns

CNT-104S has standard TCXO (uncertainty 1ppm) and 100 samples average for calibration.

External plus internal input white noise is assumed to be 0.5 mV rms.

Random uncertainty

Resolution: 10 ps rms (single shot), 1 ps rms (averaged over 100 samples)

Noise trigger error: 0.9 ps rms (single shot), 90 fs rms (averaged over 100 samples)

Systematic uncertainty

Timebase error: <100ps (100µs * 1ppm)

Trigger level timing error: <18 ps without calibration, <10 ps with calibration

Channel-to channel offset: 100 ps rms without calibration, <10 ps with calibration.

TOTAL Uncertainty is in the order of 100 ps

SOME THEORY

Uncertainty Types and Terminology

Every measurement is associated with an uncertainty of the measured value. The measurement result can vary due to variations of the test object, environmental factors (e.g. temperature), system factors (e.g. noise, cable reflections), and uncertainty of the measurement instrument itself.

The uncertainty components are according to international praxis divided into two groups

- **Type A or Random:**
  - Those which are evaluated by Statistical Methods and are subject to Random Effects
- **Type B or Systematic:**
  - Those which are evaluated by other means and may be subject to Systematic Effects

Example of Type A (Random) uncertainties

Quantization error (resolution)

Instrumental Noise

Measurement "System" Noise

Repeatability

These uncertainties are combined using a Root-Mean-Square (RMS) approach. When you combine several random variables, the combined result approaches a normal distribution (Gaussian distribution):

Combining Random Uncertainties

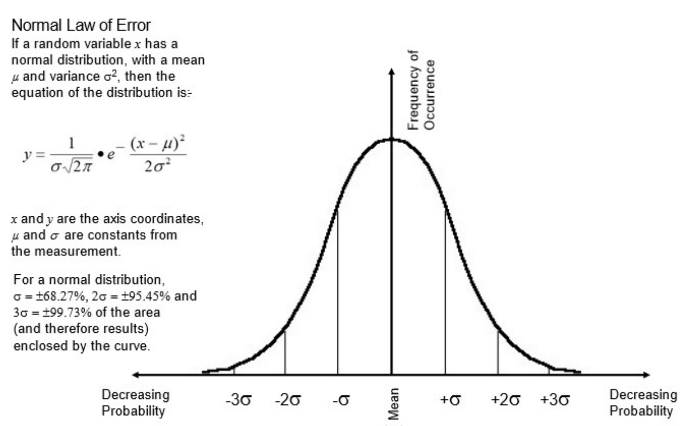
(s = standard deviation)

$u_R = \sqrt{(s_1^2+s_2^2+s_3^2+...)}$

For a normal distribution, σ=±68.27%, 2σ=±95.45% and 3σ=±99.73% of the area

(and therefore results) enclosed by the curve.

A random parameter specified as "rms" is assumed to have a "one-sigma confidence" meaning that 68% of all results are inside the rms-spec.



Systematic (Type B) uncertainties

Uncorrected Errors

Residuals after Error Correction

Stability with Time and Temperature

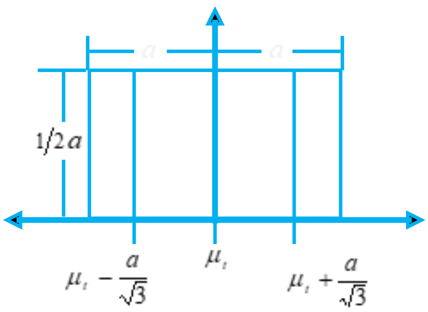
Linearity and Power-Related Errors

Instrument Setting Errors

Operator Bias

Type B errors in datasheets are normally given as limit values where the manufacturer guarantees that no value is outside the fixed limits. For a correct uncertainty handling these limits should be converted to standard deviation

For most systematic uncertainties you can assume rectangular distribution within limits ±α , The associated standard deviation for a rectangular distribution, where you assume that all values inside the limits have equal probability, α/√3.



For a Rectangular Distribution, there is an equal probability of a sample occurring anywhere within the Rectangular Probability Limits.

Combining Random Uncertainties

(α = limit value)

$u_S=\sqrt{((a_1^2+a_2^2+a_3^2+\ldots)/3)}$

Combining random and systematic uncertainty - summary

- Identify all known sources of error
- Separate into Types A (Random) and B (Systematic)
- Apply corrections if practicable (Type B)
- Calculate Type A Standard Uncertainty $u_R=\sqrt{(s_1^2+s_2^2+s_3^2+\ldots)}$
- Calculate Type B Standard Uncertainty $u_S=\sqrt{((a_1^2+a_2^2+a_3^2+\ldots)/3)}$
- Combine Random and systematic uncertainties $U=\sqrt{(u_R^2+u_S^2)}$
- Calculate Expanded Uncertainty for k = 2 or 2×U, according to standard metrology practise.